HORNSBY GIRLS HIGH SCHOOL

Mathematics

Year 12 Higher School Certificate

Trial Examination Term 3 2019

STUDENT NUMBER: ____________________

General Instructions

- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black pen
- NESA-approved calculators and drawing templates may be used
- A reference sheet is provided separately
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination room

Total marks – 100

Section I Pages 3 – 6

10 marks

Attempt Questions 1 – 10

Answer on the Objective Response Answer Sheet provided

Section II Pages 7 – 15

90 marks

Attempt Questions 11 – 16

Start each question in a new writing booklet

Write your student number on every writing booklet

Question	*1-10*	*11*	*12*	*13*	*14*	*15*	*16*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

10 Marks.

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the Multiple Choice Answer Sheet to complete this section.

---

1. 5.9974932 rounded correct to 3 significant figures is

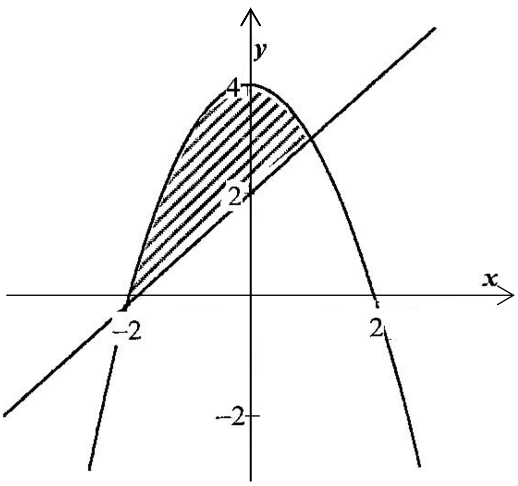
   A. 5.99

   B. 6.00

   C. 5.997

   D. 5.998

2. Which pair of inequalities represents the shaded region?



   A.
$$\begin{cases} y \le x+2 \\ y \le 4-x^2 \end{cases}$$

   B.
$$\begin{cases} y \le x+2 \\ y \ge 4-x^2 \end{cases}$$

   C.
$$\begin{cases} y \ge x+2 \\ y \ge 4-x^2 \end{cases}$$

   D.
$$\begin{cases} y \ge x+2 \\ y \le 4-x^2 \end{cases}$$

3. Simplify $\dfrac{x^2-5xy}{x^2-25y^2}$

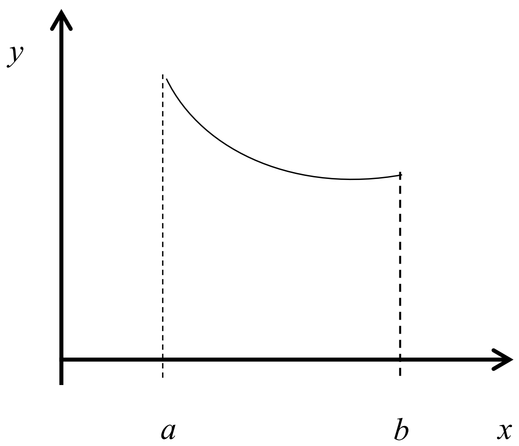
A. $\dfrac{x}{x-5y}$

B. $\dfrac{x}{x+5y}$

C. $\dfrac{1-x}{1-5y}$

D. $\dfrac{x-5y}{x+25y}$

4. For the function $y=f(x)$, $a<x<b$ graphed below:



which of the following is true?

A. $f'(x)>0$ and $f''(x)>0$

B. $f'(x)>0$ and $f''(x)<0$

C. $f'(x)<0$ and $f''(x)>0$

D. $f'(x)<0$ and $f''(x)<0$

5. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

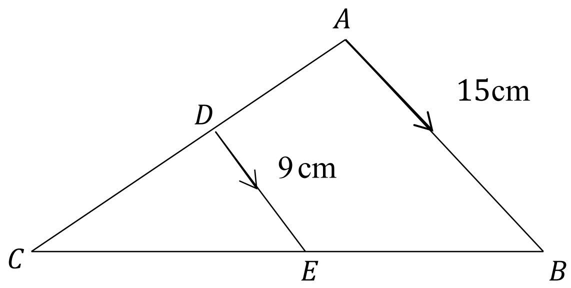
A. $k \geq -3$

B. $k \leq -3$

C. $k \geq 3$

D. $k \leq 3$

6. In the diagram below, ABC is a triangle and $AB \parallel DE$.



Given that $AB = 15$ cm, $DE = 9$ cm and $BE = 6$ cm, what is the value of BC?

A. 3.6 cm

B. 6 cm

C. 9 cm

D. 15 cm

7. What is the solution of the equation $\cos x(\tan x - 1) = 0$ for $0 \leq x \leq \pi$

A. No solution

B. $x = \dfrac{\pi}{4}$ only

C. $x = \dfrac{\pi}{2}$ only

D. $x = \dfrac{\pi}{4}$ or $x = \dfrac{\pi}{2}$

8. What is the angle of inclination of the line $3x+2y=7$ with the positive direction of the $x-$axis?

A. $33°41'$

B. $56°19'$

C. $123°41'$

D. $146°19'$

9. A particle is moving in a straight line. At time t seconds its displacement from a fixed point O on the line is $x=t^2-2t$ metres. What distance is travelled by the particle in the first 3 seconds of its motion?

A. 3 meters

B. 4 metres

C. 5 metres

D. 6 metres

10. A population is declining exponentially. After time t years the number of individuals in the population is given by $N(t)=1000e^{-0\cdot 1t}$. What percentage of the population remaining at the start of the n^{th} year is lost during that year?

A. $100(1-e^{-0\cdot 1})\%$

B. $100e^{-0\cdot 1}\%$

C. $\dfrac{100}{n}(1-e^{-0\cdot 1})\%$

D. $\dfrac{100}{n}e^{-0\cdot 1}\%$

End of Section I

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours 45 minutes for this section

Begin each question in a new writing booklet, indicating the question number.

Extra writing booklets are available

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Rationalise the denominator of $\dfrac{5}{3+\sqrt{2}}$. **2**

(b) Find the length of a circular arc in a sector of radius 4 cm subtending an angle of $\dfrac{2\pi}{3}$ radians. **1**

(c) Find $\int (1-5x)^3 \, dx$ **1**

(d) Differentiate $\dfrac{x}{\cos x}$ **2**

(e) Differentiate $x^2 e^{3x}$ **2**

(f) Find the focus and directrix of the parabola $8y = x^2 - 4x - 4$. **3**

(g) Solve $|3x-2| \le 1$ **2**

(h) Find the domain of the function $f(x) = \dfrac{1}{\sqrt{x^2 - 4}}$ **2**

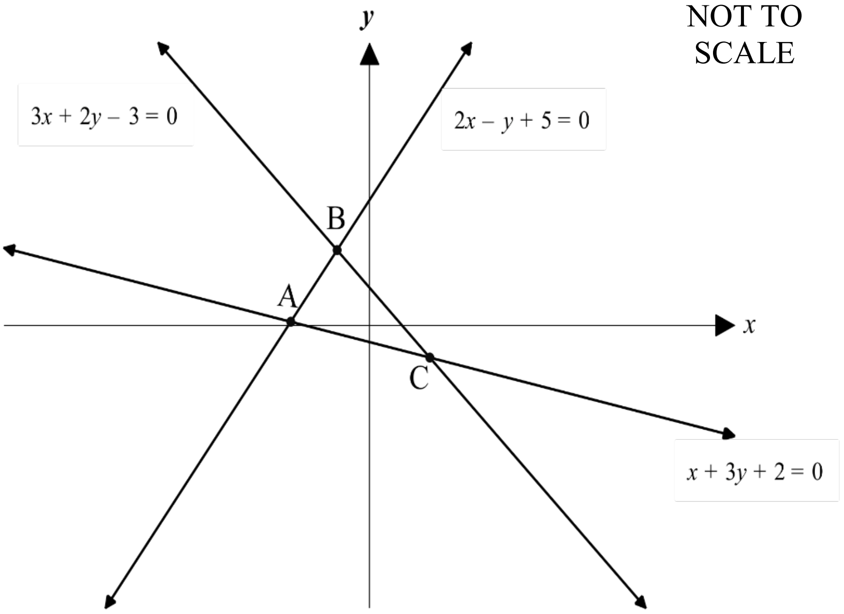
Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Find the equation of the normal to the curve $y = 2\ln(x+1)$ at $x = 0$. **2**

(b) For a particular series the sum to n terms is given by $S_n = 2^{n+1} + n^2$. **2**
What would be the 11^{th} term of this series?

(c) In the diagram below, the lines $2x - y + 5 = 0$ and $3x + 2y - 3 = 0$ intersect at B and $x + 3y + 2 = 0$ is the line AC.

NOT TO SCALE



(i) Find the coordinates of B. **1**

(ii) Find the perpendicular distance from B to AC. Leave your answer as a surd. **2**

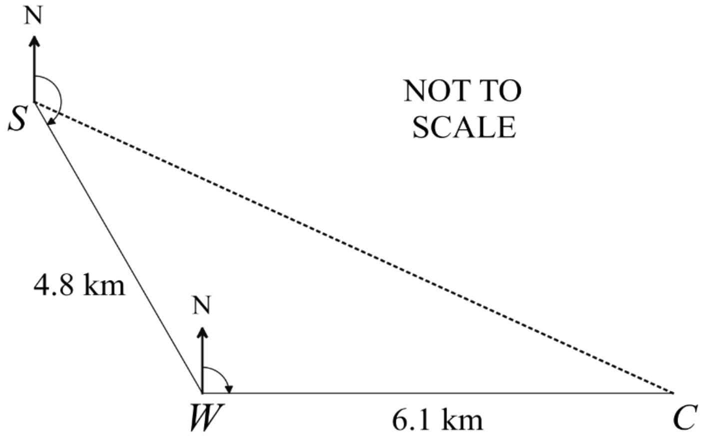
(iii) State the equation of the circle with centre B and having AC as a tangent to the circle. **1**

Question 12 continues on page 9

Question 12 (continued)

(d) Wendy sets out on a bushwalk leaving the campsite (S) on a bearing of 150° T for 4.8 km until she reaches the waterfall (W).

She has a swim and then sets out walking due east for 6.1 km until she arrives to the edge of the canyon (C).

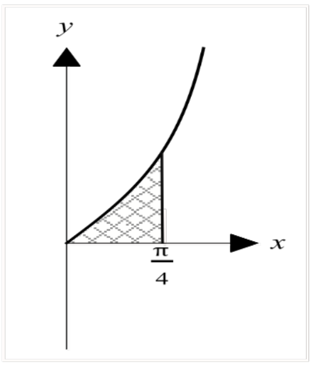


(i) What is the size of $\angle SWC$? **1**

(ii) Show that the distance of the canyon (C) from the campsite (S) is 9.5 km, correct to the nearest 100 metres. **1**

(iii) Hence, or otherwise, find the bearing of the campsite site (S) from the Canyon (C), to the nearest degree. **2**

(e) The diagram below shows the region bounded by $y = \tan x$, the line $x = \frac{\pi}{4}$ and the x-axis. **3**



NOT TO
SCALE

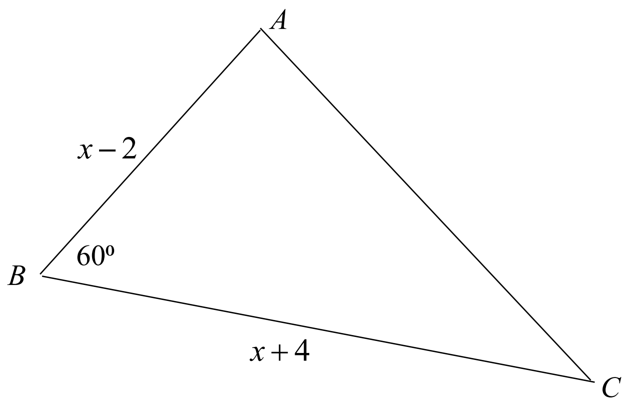
The region is rotated about the x-axis to form a solid.

Find the exact volume of the solid formed.

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) The area of ΔABC is $18\sqrt{3}\ cm^2$. Calculate the value of x. **3**



NOT TO SCALE

(b) Consider the curve $y = 4x^3 - 24x^2 + 8$

(i) Find the stationary points of the curve and determine their nature. **4**

(ii) Sketch the curve, labelling the stationary points. **2**

(iii) Hence, or otherwise, find the values of x for which $\dfrac{dy}{dx}$ is negative. **1**

(c) By letting $m = \ln x$, solve for x: $[\ln x]^2 - \ln x^3 - 4 = 0$ **2**

(d) The rate at which water flows into a bathtub is given by **3**

$$\frac{dV}{dt} = \frac{12t}{1+3t^2}$$

where V is the volume of water in the bathtub in litres and t is the time in seconds.

Initially the bathtub is empty.

Find the exact amount of water in the bathtub after 5 seconds.

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Sketch the curve $y = 3 + 2\cos \pi x$ for $0 \le x \le 4$. **3**

(b) (i) Find the exact value of $\int_0^1 e^x \, dx$ **1**

(ii) Using the trapezoidal rule with 3 function values, find an approximation to the integral $\int_0^1 e^x \, dx$, leaving your answer in exact form. **2**

(iii) Using parts (i) and (ii), show that $\sqrt{e} \approx \dfrac{3e-5}{2}$ **1**

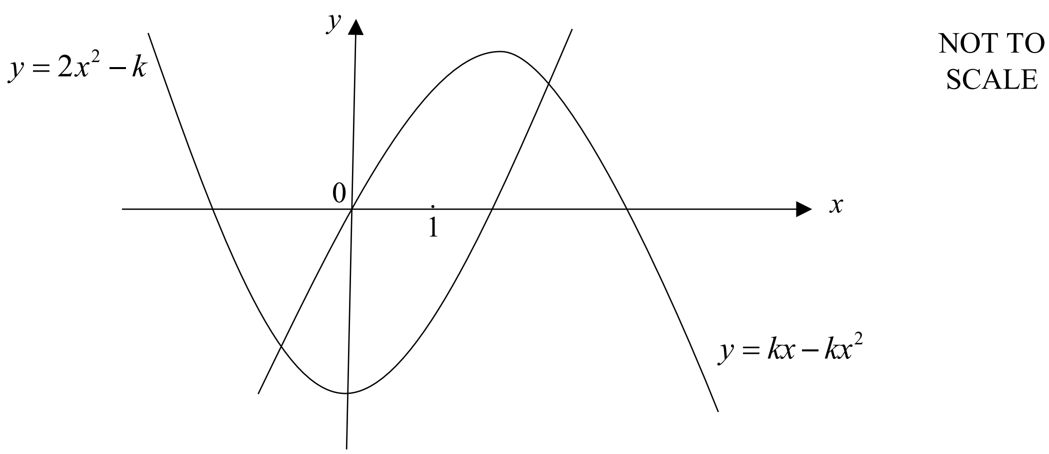
(c) The number of bacteria present in a culture is given by $N(t) = Ae^{kt}$, where A and k are constants and t is the time in minutes.

(i) Show that $N(t)$ satisfies $\dfrac{dN}{dt} = kN$ **1**

(ii) If it takes 5 minutes for the bacteria to quadruple, show that $k = 0.2773$, correct to 4 significant figures. **2**

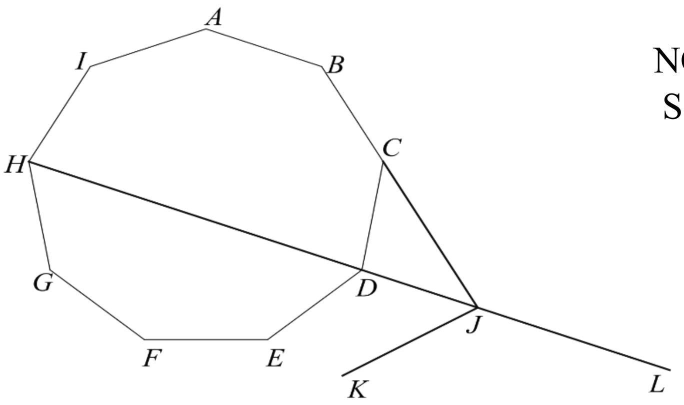
(iii) Find the rate of change of the bacteria after one hour given that the initial amount of bacteria is 6.2×10^6. Express your answer in scientific notation correct to three significant figures. **2**

(d) The area between the curves $y = kx - kx^2$ and $y = 2x^2 - k$, for $0 \le x \le 1$ where $k > 1$, is 4 square units. Find the value of k. **3**



Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) In the diagram, $ABCDEFGHI$ is a regular nonagon and BC is produced to J, such that $DC = DJ$ and $BJ \perp KJ$. HD is produced to L passing through J.



(i) Find the size of $\angle BCD$. **1**

(ii) Find the size of $\angle KJL$ giving reasons. **2**

(b) Two particles A and B, initially at the origin move along the x-axis. Their velocities v are in metres per second at time t.

Particle A has displacement given by $x = \dfrac{t^4}{4} - t^3 + 2t^2 + 3t$.

(i) Find the velocity v of particle A as a function of time. **1**

Particle B has velocity given by $v = 4t + 3$.

(ii) When do the two particles have the same velocity? **1**

(iii) What is the distance of the particles from the origin, when both particles meet again? **3**

(iv) Show that the acceleration is never less than $1\,\text{m/s}^2$ for particle A. **2**

.

Question 15 continues on page 13

(c) Peter borrows \$26 000 from his parents to buy a new car.
His parents agree to lend Peter the money to be repaid over a period of 5 years with regular monthly repayments of $\$M$ made at the end of each month.

Peter's parents agree to not charge Peter any interest for the first 2 years of the loan period. Thereafter, at the end of each month, interest of 2% per month is calculated on the amount owing and is charged just before each repayment.

Let $\$A_n$ be the amount owing at the end of the nth repayment.

(i) Explain why $A_{24} = 26000 - 24M$. **1**

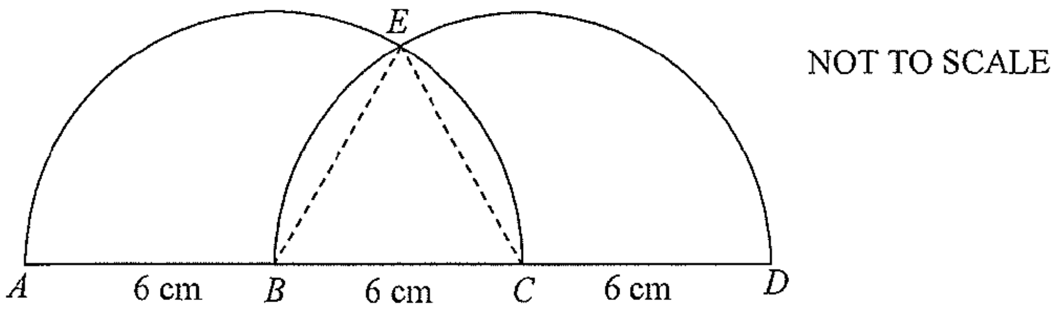
(ii) Show that $A_{26} = \left(26\,000 - 24M\right)\left(1.02\right)^2 - M\left(1 + 1.02\right)$ **1**

(iii) Find the amount of each monthly repayment. **3**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a)

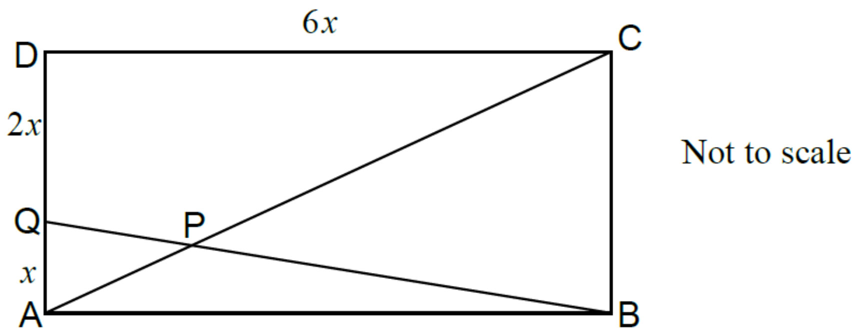


In the diagram, $ABCD$ is a straight line where $AB = BC = CD = 6$cm. **2**

The semicircles on diameters AC and BD intersect at E so that ΔBCE is equilateral.

Find in simplest exact form the area of the region common to the two semicircles.

(b) The diagram below shows rectangle $ABCD$ where $CD = 6x$, $QD = 2x$ and $QA = x$.
The line BQ meets AC at P.



Copy the diagram into your answer booklet.

(i) Prove $\Delta APQ \,|||\, \Delta CPB$ **2**

(ii) Show $CP = \dfrac{3}{4}AC$ **2**

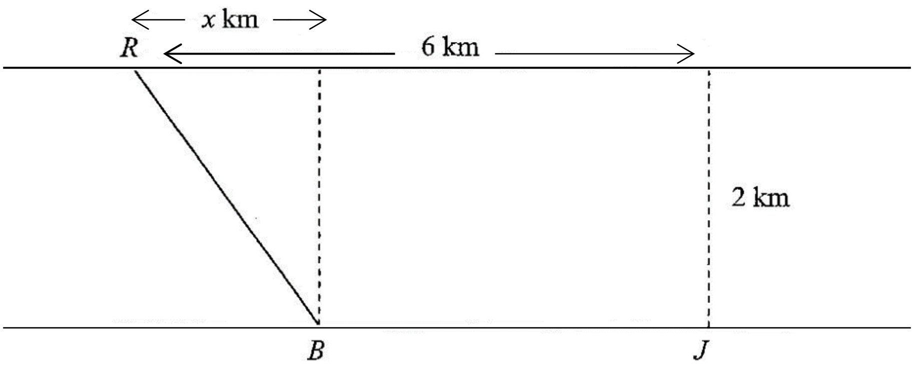
(c) Given that $|x| < 1$ and $\dfrac{1+x}{3x} = 1 + x + x^2 + \ldots$ to infinity, find the value of x . **3**

Question 16 continues on page 15

(d) Romeo (R) and Juliet (J) live on 2 parallel streets which are 2 km apart and run east-west as shown in the diagram. Juliet calls Romeo to let him know she is home. Romeo immediately leaves his house in order to get to Juliet's house as soon as possible.

Romeo has hidden a bike at point B on Juliet's street.
To get to Juliet's house, Romeo runs at a speed of 8km/h, from his house, R, through the bush to his bike, B. He then rides his bike, at a speed of 16km/h, to Juliet's house, J.

Let x km represent the distance the bike is east of Romeo's house.



(i) Show that the time taken in hours, T, for Romeo to get to Juliet's house is given by **2**

$$T = \frac{\sqrt{x^2+4}}{8} + \frac{6-x}{16}$$

(ii) Find the distance BJ, in order to minimise the time taken for Romeo to get to Juliet. **3**

(iii) Find the minimum time taken for Romeo to get to Juliet's house. **1**

End of paper

Hornsby Girls High School

Yr 12 HSC Trial Mathematics 2U 2019

1	B
2	D
3	B
4	C
5	A
6	D
7	B
8	C
9	C
10	A

1) 6·00 B

2) $y \geq 2$

$y \leq 4 - x^2$ D

3) $\dfrac{x^2 - 5xy}{x^2 - 25y^2} = \dfrac{x(\cancel{x-5y})}{(x+5y)(\cancel{x-5y})}$

$= \dfrac{x}{x+5y}$ B

4) Decreasing $\therefore f'(x) < 0$

Concave up $\therefore f''(x) > 0$ C

5) $x^2 - 6x - 3k = 0$ has real roots when $\Delta \geq 0$

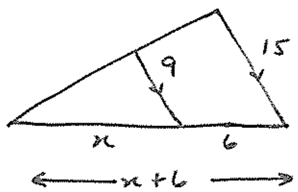
$\Delta = (-6)^2 - 4 \times 1 \times (-3k)$

$\Delta = 36 + 12k \geq 0$

$12k \geq -36$

$k \geq -3$ A

6)



$\dfrac{x}{9} = \dfrac{x+6}{15}$

$15x = 9x + 54$

$6x = 54$

$x = 9$

$x + 6 = 9 + 6 = 15$ cm D

7) $\cos x(\tan x - 1) = 0$ for $0 \leq x \leq \pi$

$\cos x = 0$ or $\tan x = 1$

$x = \frac{\pi}{2}$ $x = \frac{\pi}{4}$

but $\tan\frac{\pi}{2}$ is undefined

$\therefore x = \frac{\pi}{4}$ only B

8) $3x + 2y = 7$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

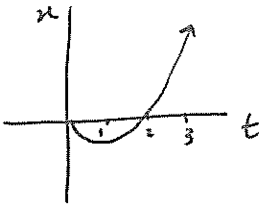
$m = -\frac{3}{2}$

$\tan\theta = -\frac{3}{2}$

$\theta = 123^\circ 41'$ C

9) $x = t^2 - 2t$

$x = t(t-2)$



when $t=0$, $x=0$

when $t=1$, $x=-1$

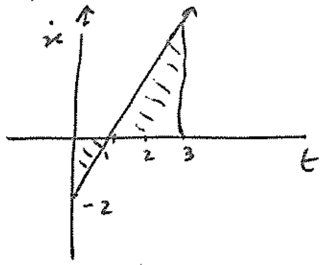
when $t=3$, $x = 9-6 = 3$

Distance travelled $= 1+4$

$= 5$ C

or $x = t^2 - 2t$

$\dot{x} = 2t - 2$



$d = \left|\int_0^1 (2t-2)\,dt\right| + \int_1^3 (2t-2)\,dt$

$= -\left[t^2 - 2t\right]_0^1 + \left[t^2 - 2t\right]_1^3$

$= -\left[1-2-(0-0)\right] + \left[9-6-(1-2)\right]$

$= -\ -1 \quad + \quad 3-(-1)$

$= \quad 1 \quad + 3 + 1$

$= 5$

or areas of triangles

10) Number of individuals at start of nth year is $N(n) = 1000e^{-0.1n}$

" " " " $(n+1)$th $N(n+1) = 1000e^{-0.1(n+1)}$

$\therefore$ Percentage lost $= \dfrac{N(n) - N(n+1)}{N(n)} \times 100\%$

$= \dfrac{1000e^{-0.1n} - 1000e^{-0.1n - 0.1}}{1000e^{-0.1n}} \times 100\%$

$= \dfrac{\cancel{1000e^{-0.1n}}\left[1 - e^{-0.1}\right]}{\cancel{1000e^{-0.1n}}} \times 100\%$

$= (1 - e^{-0.1}) \times 100\%$ A.

Q 11

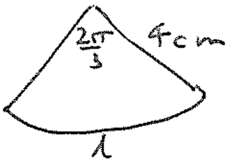
a) $\frac{5}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{15-5\sqrt{2}}{9-2}$

$= \frac{15-5\sqrt{2}}{7}$

Done well

2 1 for method

1 for accuracy

b) 

$l = r\theta$

$= 4 \times \frac{2\pi}{3}$

$= \frac{8\pi}{3}$ cm

$\doteqdot 8.37758041$ cm

Done well

1 exact form was preferable.

c) $\int (1-5x)^3 dx = \frac{(1-5x)^4}{4\times(-5)} + c$

$= \frac{(1-5x)^4}{-20} + c$

1 Mostly done well. Some student did not divide by the derivative of $(1-5x)$

Most students remembered $+c$,

d) $\frac{d}{dx} \frac{x}{\cos x} = \frac{\cos x \times 1 - x \times (-\sin x)}{\cos^2 x}$

$= \frac{\cos x + x\sin x}{\cos^2 x}$

2 1. for quotient rule

1. for correct differentiation

e) $\frac{d}{dx} x^2 e^{3x} = e^{3x} \times 2x + x^2 \times 3e^{3x}$

$= 2xe^{3x} + 3x^2e^{3x}$

or $xe^{3x}(2+3x)$

2 1 for product rule

1 for correct differentiation

f) $8y = x^2 - 4x - 4$

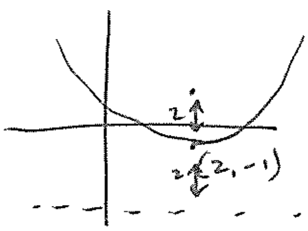
$8y = x^2 - 4x + 4 - 8$

$8y = (x-2)^2 - 8$

$8y + 8 = (x-2)^2$

$8(y+1) = (x-2)^2$

Which is a parabola with vertex $(2,-1)$; $a = 2$



$\therefore$ focus $(2,1)$

directrix $y = -3$

This question was poorly done. Some students did not recognise it as a parabola and did not complete the square

1. for rewriting in the form $(x-h)^2 = 4a(y-k)$

1 for correct focus

1 for correct directrix

Q 11) g) $|3x-2| \leq 1$

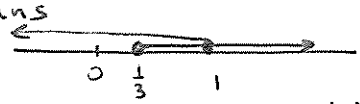
$-1 \leq 3x-2 \leq 1$ 1 for dealing with the absolute value

$1 \leq 3x \leq 3$

$\frac{1}{3} \leq x \leq 1$ 1 for correctly solving

Many students left their answer as $x \geq \frac{1}{3}$, $x \leq 1$

which means



graphically, which is all real solutions

Students must interpret their solution and combine their answer to $\frac{1}{3} \leq x \leq 1$

h) $f(x) = \frac{1}{\sqrt{x^2-4}}$

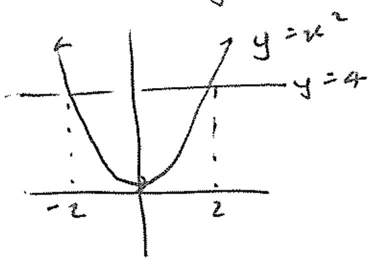
The function is defined when

$x^2 - 4 > 0$ 1

$x^2 > 4$

$x < -2, x > 2$ 1

Many students could not solve $x^2 > 4$ correctly.



Q12 a $y = 2\ln(x+1)$

$\frac{dy}{dx} = \frac{2}{x+1}$

at $x=0$, $y = 2\ln(0+1)$
$= 2\ln(1)$
$= 0$

at $x=0$, $\frac{dy}{dx} = \frac{2}{1} = 2$ ✓

$\therefore$ gradient of normal: $m = -\frac{1}{2}$

Equation of normal: $m = -\frac{1}{2}$ $(0,0)$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{2}(x - 0)$

$y = -\frac{x}{2}$ or $x + 2y = 0$ ✓

4T

b $S_n = 2^{n+1} + n^2$

1/ S10 or 11.

$T_{11} = S_{11} - S_{10}$ ✓

$= 2^{11+1} + 11^2 - (2^{10+1} + 10^2)$ 2 / an

$= 2069$ ✓

coordinates of B

i) Solve simultaneously $2x - y + 5 = 0$ ①

$3x + 2y - 3 = 0$ ②

① ×2 $4x - 2y + 10 = 0$ ③

③ + ② $7x + 7 = 0$

$7x = -7$

$x = -1$

when $x = -1$ sub into ①

$\therefore 2(-1) - y + 5 = 0$

$-2 - y + 5 = 0$

$-y = -3$

$\therefore y = 3$

$\therefore B(-1, 3)$

$B(-1, 3)$ AC: $x + 3y + 2 = 0$

ii) $d_\perp = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|1(-1) + 3(3) + 2|}{\sqrt{(1)^2 + (3)^2}}$

$= \dfrac{|-1 + 9 + 2|}{\sqrt{1 + 9}}$

$= \dfrac{10}{\sqrt{10}}$

$= \sqrt{10}$ units

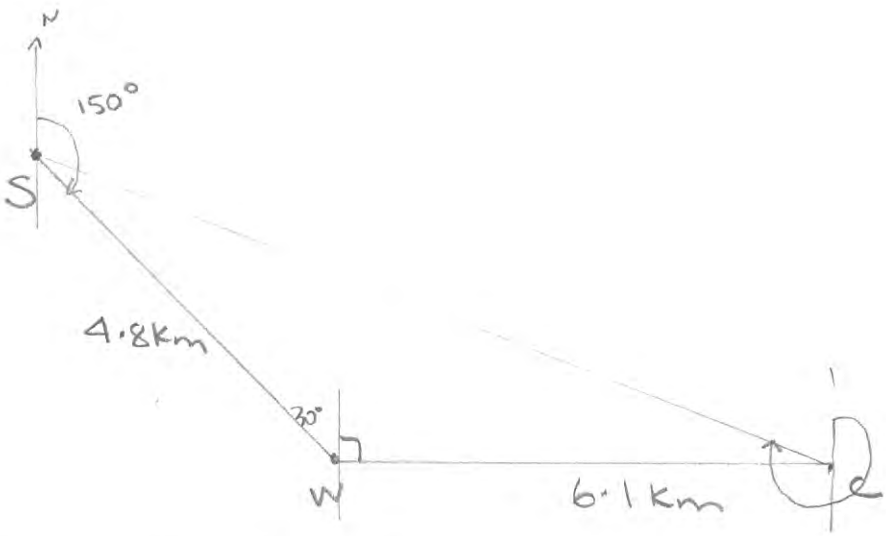
iii) Center: $B(-1, 3)$

$r = \sqrt{10}$

$(x+1)^2 + (y-3)^2 = (\sqrt{10})^2$

$(x+1)^2 + (y-3)^2 = 10$

a



Cointerior ∠'s $150° + 30°$

i) $\angle SWC = 30° + 90°$
$= 120°$

ii) $SC^2 = SW^2 + WC^2 - 2 \times SW \times WC \times \cos 120°$
$= 4.8^2 + 6.1^2 - 2 \times 4.8 \times 6.1 \times \cos 120°$
$= 89.53$

$\therefore SC = \sqrt{89.53}$ $SC > 0$
$= 9.462...$
$\doteqdot 9.5$ (as required)

∴ Distance of canyon from campsite is 9.5km

iii) Bearing is $270° + \angle SCW$

Finding $\angle SCW$:

$$\frac{\sin \angle SCW}{4.8} = \frac{\sin 120°}{9.5}$$

$$\sin \angle SCW = \frac{4.8 \times \sin 120°}{9.5} = 0.4375$$

$\therefore \angle SCW = \sin^{-1}(0.4375)$
$= 25.94$
$\doteqdot 26°$

∴ Bearing
$= 270 + 26$
$= 296°T$

e. $V = \pi \int_0^{\frac{\pi}{4}} (\tan x)^2 \, dx$

$= \pi \int_0^{\pi/4} \tan^2 x \, dx$

$= \pi \int_0^{\pi/4} (\sec^2 x - 1) \, dx$

$= \pi \left[\tan x - x \right]_0^{\pi/4}$

$= \pi \left[\left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) - (\tan 0 - 0) \right]$

$= \pi \left[1 - \frac{\pi}{4} - 0 \right]$

$= \pi - \frac{\pi^2}{4}$

$= \frac{4\pi - \pi^2}{4} \; u^3$

Question 12 Feedback

(a) Generally well done. Students used both $y - y_1 = m(x - x_1)$ and $y = mx + b$ successfully. Main error was incorrectly evaluating $\log(1) = 0$.

(b) Students were successful is they saw the link between finding S_{11} and S_{10} and finding T_{11}.

One mark was awarded for successfully using the sum formula, or trying to generate a pattern to find the term using successive sums/terms.

(c)

(i) Many students made algebraic/arithmetic errors. It would be worth any time that points are found by solving simultaneous that the results are tested in both equations to ensure they are correct. If you end up with horrible fractional coordinates, it is definitely worth doing that check.

(ii) Students were generally successful here. It is important to show the substitution into the formula – it is advised to write the formula and then show the substitution so that errors can be following through and marks awarded. Generally, students should rationalise distances where easy. Units should be given for a distance.

(iii) Students successfully used the previous two parts to complete this question, even with errors in previous parts. Main errors were forgetting squares, or arithmetic errors when squaring.

(d)

(i) Well done.

(ii) This is a show question. Formula and substitution should be shown. Calculator output before rounding should be shown, and then next line rounding as required.

(iii) Students using the sine rule were more successful here. Some students struggled with finding $\angle SCW$ and then applying the result to get the correct bearing.

(e) The main errors with this part were:

- Leaving pi off the volume formula.
- Not squaring the function
- Integrating the function to get a logarithmic function
- Attempting to use reverse chain rule on tan squared.
- Using integration by substitution where it is not in 2 unit syllabus and therefore probably not the easiest approach.
- Incorrectly replacing $\tan^2 x = \sec^2 x - 1$,
- Incorrectly adding unlike terms involving π .
- Not including cubic units.

Q13.

(a) $18\sqrt{3} = \frac{1}{2}(x-2)(x+4)\sin 60^\circ$

$18\sqrt{3} = \frac{1}{2}(x-2)(x+4)\frac{\sqrt{3}}{2}$

$72 = (x-2)(x+4)$ 1 NB Many students used the quadratic formula to factorise this quadratic

$x^2+2x-8=72$

$x^2+2x-80=0$

$(x+10)(x-8)=0$ 1

$\therefore x=-10,\ x=8$

no soln.

$\therefore x=8$ 1

(b) i) $y=4x^3-24x^2+8$

$y' = 12x^2-48x = 0$ for stat. pts

$0 = 12x(x-4)$

$\therefore x=0,\ x=4$ 1 knowing to put $y'=0$

when $x=0$, $y=8$

when $x=4$, $y=4(4)^3-24(4)^2+8$

$y=-120$

$\therefore$ stat. pt at $(0,8)$ & $(4,-120)$ 1 obtaining y's

$y'' = 24x-48$ 1 knowing how to distinguish between max & min pt

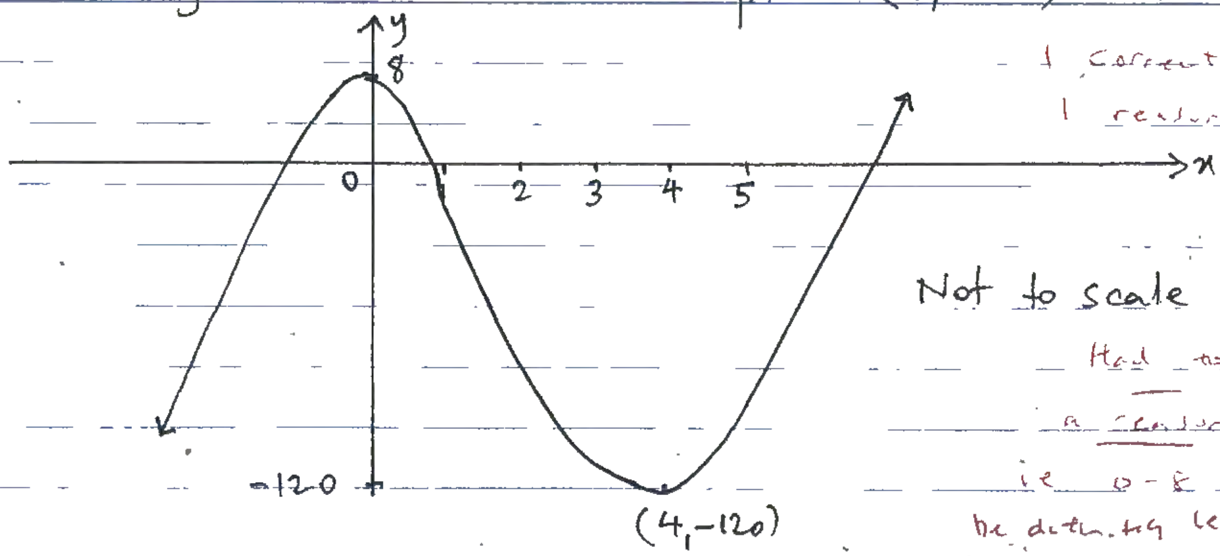
when $x=0$, $y''=24(0)-48$

$y''=-48<0 \ \therefore$ max. turn. pt at $(0,8)$

when $x=4$, $y''=24(4)-48$ 1 – finding actual pts and distinguishing them

$y''=48>0 \ \therefore$ min. turn. pt at $(4,-120)$

ii)



1 correct shape

1 reasonable scale

Had to have a reasonable scale i.e. 0–8 should be distance less than distance 0 to −120.

Q13 cont

iii) $\frac{dy}{dx} < 0$ for $0 < x < 4$

c) $[\ln x]^2 - \ln x^3 - 4 = 0$

$[\ln x]^2 - 3\ln x - 4 = 0$

let $m = \ln x$

$m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

$\therefore m = 4, \quad m = -1$

$\ln x = 4, \quad \ln x = -1$

$x = e^4, \quad x = e^{-1}$

d) $\frac{dV}{dt} = \frac{12t}{1+3t^2} \quad dt$

$V = \int_0^5 \frac{12t}{1+3t^2} \, dt$

$V = 2\int_0^5 \frac{6t}{1+3t^2} \, dt$

$V = \left[2\ln(1+3t^2)\right]_0^5$

$V = 2\ln(1+3(5)^2) - 2\ln(1+0)$

$V = 2\ln 76 - 0$

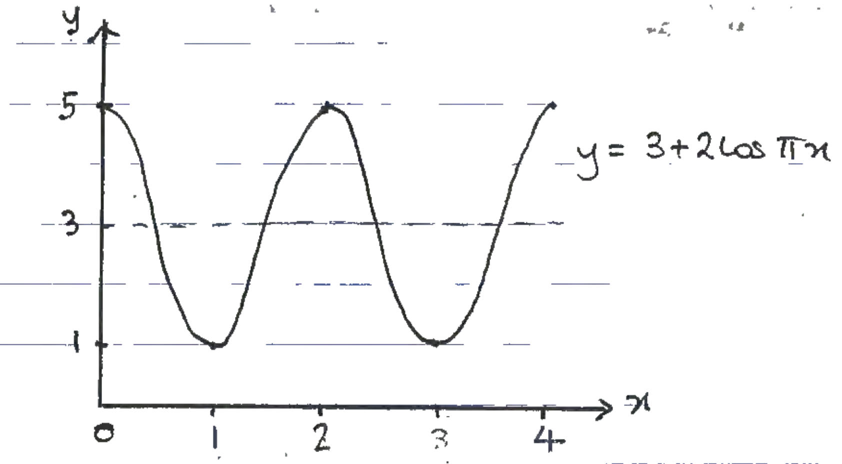
$V = 2\ln 76$

Q14.

a) $a = 2$, period $= \frac{2\pi}{\pi}$

period $= 2$

1 mark for amplitude
1 mark for period
1 mark for shape of graph/graph itself according to ampl. & period



3

b) i) $\int_0^1 e^x \, dx$

$= \left[e^x\right]_0^1$

$= e^1 - e^0$

$= e - 1$ 1

ii) $h = \frac{1-0}{2} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1
$f(x)$	1	$e^{\frac{1}{2}}$	e^1

1 mark for values

$A \doteqdot \frac{\frac{1}{2}}{2}\left[1 + e + 2e^{\frac{1}{2}}\right]$

$A \doteqdot \frac{1}{4}\left[1 + e + 2\sqrt{e}\right] u^2$ 2

1 mark for subst. into trapezoidal rule.

iii) $e - 1 \approx \frac{1}{4}\left[1 + e + 2\sqrt{e}\right]$

$4e - 4 \approx 1 + e + 2\sqrt{e}$

$3e - 5 \approx 2\sqrt{e}$

$\therefore \sqrt{e} \approx \frac{3e-5}{2}$ 1

Q14 cont

c) i) $N(t) = Ae^{kt}$

$\frac{dN}{dt} = kAe^{kt}$

$= kN$ as $N = Ae^{kt}$ 1

ii) Let $N(t) = 4A$ as the initial amount is quadrupled.

$4A = Ae^{5t}$

$4 = e^{5t}$

$5t = \ln 4$ | 1 mark for correct answer

$t = \frac{\ln 4}{5}$ | 1 mark for sig. figs

$t = 0.2773$ /2

iii) $t = 60$

$\frac{dN}{dt} = kAe^{kt}$ | 1 mark for subst

$= 0.2773 \times 6.2 \times 10^{6} \times e^{0.2273(60)}$ | 1 mark for scientific notation

$\frac{dN}{dt} = 2.89 \times 10^{13}$ bacteria/minute. /2

d) $4 = \int_0^1 (kx - kx^2) - (2x^2 - k)\ dx$

$4 = \int_0^1 (kx - kx^2 - 2x^2 + k)\ dx$ | 1 mark for expressing area between two curves

$4 = \left[\frac{kx^2}{2} - \frac{kx^3}{3} - \frac{2x^3}{3} + kx\right]_0^1$

$4 = \left[\left(\frac{k}{2} - \frac{k}{3} - \frac{2}{3} + k\right) - 0\right]$ | 1 mark for integration & subst.

$4 = \frac{k}{2} - \frac{k}{3} - \frac{2}{3} + k$

$4\frac{2}{3} = k\left(\frac{1}{2} - \frac{1}{3} + 1\right)$ | 1 mark for correct value of k.

$4\frac{2}{3} = k\left(\frac{7}{6}\right)$

$\therefore k = 4\frac{2}{3} \div \frac{7}{6}$

$\therefore k = 4$

Question 15: HSC Trial 2019.

a. Regular nonagon ∴ 9 Sides

i) $\angle BCD = \dfrac{(9-2)\times 180^\circ}{9}$

$= 140^\circ$

1

Generally done well.

ii) $\angle BCD + \angle DCJ = 180^\circ$ (angles on straight line BCJ)

$\therefore 140^\circ + \angle DCJ = 180^\circ$

$\angle DCJ = 180^\circ - 140^\circ$

$= 40^\circ$

DC = DJ given

$\angle DCJ = \angle DJC$ (equal angles opposite equal sides in ΔCDJ)

$\therefore \angle DJC = 40^\circ$

$\angle DJC + \angle KJD = 90^\circ$ ($BJ \perp KJ$)

Some gave reason given.

$\therefore 40^\circ + \angle KJD = 90^\circ$

$\angle KJD = 90^\circ - 40^\circ$

$= 50^\circ$

$\angle DJK + \angle KJL = 180^\circ$ (angles on straight line DJL)

$50^\circ + \angle KJL = 180^\circ$

$\angle KJL = 180^\circ - 150^\circ$

$= 130^\circ$

2

Better Setting out required by many Students.

-1 if only complementary / supplementary given as a reason

1 - correct answer

1 - reasoning along the way

- ignored poor setting out
- ignored if missed a reason.

b) i) $x_A = \frac{t^4}{4} - t^3 + 2t^2 + 3t$.

$v_A = \frac{4t^3}{4} - 3t^2 + 4t + 3$

$= t^3 - 3t^2 + 4t + 3$ (1)

ii) $v_B = 4t + 3$

Same velocity Solving $v_A = v_B$

$t^3 - 3t^2 + 4t + 3 = 4t + 3$

$\therefore t^3 - 3t^2 = 0$

$t^2(t-3) = 0$

$\therefore t = 0$ or $t = 3$ (1)

* Need both $t=0$ & $t=3$ to get the mark.

$\therefore$ The particles have the same velocity initially and after 3 seconds.

iii) Find time when both particles meet again: 3marks.

$x_A = \frac{t^4}{4} - t^3 + 2t^2 + 3t$.

$x_B = \frac{4t^2}{2} + 3t + c$

at $t=0, x=0$, $\therefore c = 0$

$x_B = 2t^2 + 3t$. (1)

Solving $x_A = x_B$

$\frac{t^4}{4} - t^3 + 2t^2 + 3t = 2t^2 + 3t$.

$\frac{t^4}{4} - t^3 = 0$ (1)

$t^3\left(\frac{t}{4} - 1\right) = 0$

$\therefore t = 0$ or $\frac{t}{4} - 1 = 0$ $t = 4$.

$\therefore$ The particles meet again after 4 seconds.

When $t=4$ Sub into x_A or x_B.

$\therefore x_B = 2t^2 + 3t$

$= 2(4)^2 + 3(4)$

$= 2\times16 + 12$

$= 44\,m.$

mark was given for $x = 44\,m$.

$\therefore$ Distance from origin of the particles when they meet again is $44\,m$. (1)

* Many have misread question and used time from part (ii)
* Some did not continue to find the distance from the origin.

iv) $V_A = t^3 - 3t^2 + 4t + 3$

$\therefore a_A = 3t^2 - 6t + 4$ (1) — 1 mark for finding acceleration

$= 3(t^2 - 2t + 1) - 3 + 4$

$= 3(t-1)^2 + 1$

$\therefore$ for $t \geq 0$ $(t-1)^2 \geq 0$ (1)

$3(t-1)^2 \geq 0$

$\therefore 3(t-1)^2 + 1 \geq 1$

Hence the acceleration is never less than $1\,m/s^2$ for $t \geq 0$. OR,

* There were other methods that were acceptable.

Alternative method: $V_A = t^3 - 3t^2 + 4t + 3$

$a_A = 3t^2 - 6t + 4$

min turning point when at $t = \frac{-b}{2a} = \frac{6}{2(3)} = \frac{6}{6} = 1$

Some stop at $t=1$ and do not show that at $t=1$ $a=1$.

when $t=1$ $a = 3(1)^2 - 6(1) + 4(1)$

$= 1$ $(1,1)$

$a = 3 > 0$

$\therefore$ concave up curve $\therefore a \geq 1$

c) i) Explain why $A_{24} = 26000 - 24M$.

For two years every month Peter pays \$M taken away from the amount borrowed of \$26000.
$\therefore$ 24 months here $24 \times \$M = 24M$
$\therefore$ The amount owing of \$26000 $- 24M$. or
here $A_1 = 26000 - M$

* in words or mathematically. need to mention the principal of \$26000 (ie amount owing). 1

$A_2 = 26000 - 2M$

$A_3 = 26000 - 3M$

$\vdots$

$A_{24} = 26000 - 24M$.

ii) Show that $A_{26} = (26000 - 24M)(1.02)^2 - M(1 + 1.02)$

$A_{25} = A_{24}(1.02) - M$

$= (26000 - 24M)(1.02) - M$ * Show 1

$A_{26} = A_{25}(1.02) - M$

$= [(26000 - 24M)(1.02) - M](1.02) - M$

Need to show at least one of these lines.

$= (26000 - 24M)(1.02)^2 - M(1.02) - M$

$= (26000 - 24M)(1.02)^2 - M(1 + 1.02)$ as required.

iii. 5 years = 60 monthly repayments.

Loan is repaid when $A_{60} = 0$

$$\therefore \quad A_{60} = (26000 - 24M)(1.02)^{36} - M\underbrace{(1 + 1.02 + 1.02^2 + \dots 1.02^{35})}_{\text{Sum of a G.S.}}.$$

1

$$\therefore \quad (26000 - 24M)(1.02)^{36} - M\left[\frac{1.(1.02^{36} - 1)}{1.02 - 1}\right] = 0$$

$$26000(1.02)^{36} - 24(1.02)^{36}M - M\left[\frac{1.02^{36} - 1}{0.02}\right] = 0$$

$$-M\left[24(1.02)^{36} + \left[\frac{1.02^{36} - 1}{0.02}\right]\right] = -26000(1.02)^{36}$$

1

$$\therefore \quad M = \frac{26000(1.02)^{36}}{\left[24(1.02)^{36} + \left[\frac{1.02^{36} - 1}{0.02}\right]\right]}$$

$= \$525.3709\ldots$

$\doteqdot \$525.37$.

$\therefore$ Each repayment is $\$525.37$. 1

Q16. a) $A_{sector} = \frac{60}{360} \times \pi r^2$

$= \frac{1}{6} \times \pi \times 36$

$= 6\pi$

$A_{triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 60$

$= 18 \times \frac{\sqrt{3}}{2}$

$= 9\sqrt{3}$

(1)

Lots of numerical errors

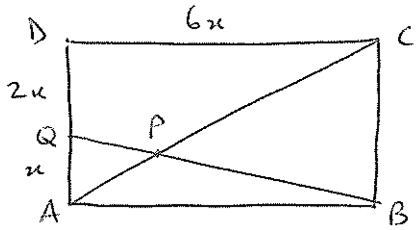
$A_{segment} = 6\pi - 9\sqrt{3}$

$A_{common\ region} = A_{sector} + A_{segment}$ or $A_{triangle} + 2 \times A_{seg.}$

$= 6\pi + 6\pi - 9\sqrt{3}$

$= 12\pi - 9\sqrt{3}\ cm^2$ (1)

b)



(i) In ΔAPQ and ΔCPB

$\angle APQ = \angle CPB$ vertically opposite angles

$\angle PQA = \angle PBC$ alternate angles on $AQ \parallel BC$ since AQ and BC are the opposite sides of a rectangle

Many students did not explain this

$\therefore \Delta APQ\ |||\ \Delta CPB$ equiangular (2)

(ii) $BC = AD$ opposite sides of a rectangle

$= QA + AD$

$= x + 2x$

$BC = 3x$

$\frac{CP}{AP} = \frac{BC}{AQ} = \frac{3x}{x} = 3$ corresponding/matching sides of similar triangles are proportional

$\therefore \frac{CP}{AP} = 3$

$CP = 3 \times AP \therefore AP = \frac{1}{3} CP$ (1)

$AC = AP + CP$

$= \frac{1}{3} CP + CP$

$AC = \frac{4}{3} . CP$

$\frac{3}{4} AC = CP$ (1)

16) c) $\frac{1+x}{3x} = 1 + x + x^2 + \ldots$

$a = 1 \ , \ r = x$

$\frac{1+x}{3x} = \frac{1}{1-x}$ 1. for finding the limiting sum

$1 - x^2 = 3x$

$0 = x^2 + 3x - 1$

$x = \frac{-3 \pm \sqrt{9 + 4 \times 1 \times 1}}{2}$

$x = \frac{-3 \pm \sqrt{13}}{2}$

$x \doteqdot \frac{0.6055}{2} \ , \ -\frac{6.6055..}{2}$ 1. for solving

but $|x| < 1$ for limiting sum to exist

$\therefore x = \frac{-3+\sqrt{13}}{2}$ only 1. for discarding $x = -3.3$

d) (i) $RB^2 = x^2 + 4$

$RB = \sqrt{x^2+4}$ km at 8 km/h 1 for distance RB

and $T = \frac{D}{S}$

$T_{RB} = \frac{\sqrt{x^2+4}}{8}$ 1 for showing the times

$BJ = (6 - x)$ km at 16 km/h

$T_{BJ} = \frac{6-x}{16}$

$\therefore T = T_{RB} + T_{BJ}$

$T = \frac{\sqrt{x^2+4}}{8} + \frac{6-x}{16}$

(ii) $T = \frac{1}{8}(x^2+4)^{\frac{1}{2}} + \frac{1}{16}(6-x)$

$T = \frac{1}{8}(x^2+4)^{\frac{1}{2}} + \frac{3}{8} - \frac{x}{16}$

$\frac{dT}{dx} = \frac{1}{2} \times \frac{1}{8}(x^2+4)^{-\frac{1}{2}} \times 2x + 0 - \frac{1}{16}$

$\frac{dT}{dx} = \frac{2x}{16\sqrt{x^2+4}} - \frac{1}{16}$

Lots of students had $\frac{+5}{16}$ instead of $\frac{-1}{16}$

16) d) (iii)

Stationary points occur when $\frac{dT}{dx} = 0$

$$\frac{2x}{16\sqrt{x^2+4}} - \frac{1}{16} = 0$$

$$\frac{2x}{16\sqrt{x^2+4}} = \frac{1}{16}$$

$$2x = \sqrt{x^2+4}$$

$$4x^2 = x^2+4$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \frac{\pm 2}{\sqrt{3}}$$ but x is a distance so $x \geq 0$

$$x = \frac{2}{\sqrt{3}} \doteqdot 1 \cdot 1547$$

① for correct differentiation and equating to zero

① for $x = \frac{2}{\sqrt{3}}$ and

$BJ = 6 - x$

$= 6 - \frac{2}{\sqrt{3}}$ later in the Q.

First derivative test

x	1	$\frac{2}{\sqrt{3}}$	2
$\frac{dT}{dx}$	$\frac{2}{16\sqrt{5}} - \frac{1}{16}$ $\doteqdot -0 \cdot 0065960$	0	$\frac{4}{16\sqrt{8}} - \frac{1}{16}$ $\doteqdot 0 \cdot 02588534765 > 0$

$\backslash \quad — \quad /$

$\therefore$ minimum time taken when $x = \frac{2}{\sqrt{3}}$

① for verifying that the minimum time is achieved when $x = \frac{2}{\sqrt{3}}$.

The first derivative test is much easier and more efficient for this question.

Many students did not do this part

OR

Second derivative test (not recommended)

$$\frac{d^2T}{dx^2} = \frac{16\sqrt{x^2+4} \times 2 - 2x \times 16 \times \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \times 2x}{\left(16\sqrt{x^2+4}\right)^2}$$

$$= \frac{32\sqrt{x^2+4} - \frac{32x^2}{\sqrt{x^2+4}}}{16^2(x^2+4)}$$

$$= \frac{32(x^2+4) - 32x^2}{16^2(x^2+4)\sqrt{x^2+4}}$$

$$= \frac{32x^2 + 128 - 32x^2}{16^2(x^2+4)^{3/2}}$$

$$\frac{d^2T}{dx^2} = \frac{128}{16^2(x^2+4)^{3/2}} > 0$$

$\therefore$ minimum time taken when $x = \frac{2}{\sqrt{3}}$

16) d) (iii) $T = \dfrac{\sqrt{x^2+4}}{8} + \dfrac{6-x}{16}$ when $x = \dfrac{2}{\sqrt{3}}$

$= \dfrac{\sqrt{5\frac{1}{3}}}{8} + \dfrac{6 - \frac{2}{\sqrt{3}}}{16}$

$= \frac{1}{8} \cdot \sqrt{\frac{16}{3}} + \frac{3}{8} - \frac{2}{16\sqrt{3}}$

$= \frac{1}{8} \times \frac{4}{\sqrt{3}} + \frac{3}{8} - \frac{1}{8\sqrt{3}}$

$T = \frac{3}{8\sqrt{3}} + \frac{3}{8}$ (1). Done well

$\doteqdot 0.5915063509$ hours

$\doteqdot 35' 29.42''$

$\doteqdot$ 35 min 29 sec